Dual-Band Rat-Race Coupler with Arbitrary Power-Split Ratios

Kuo-Sheng Chin, Yen-Hsiu Wei, Ting-Yi Lin, and Chih-Chun Chang
Department of Electronic Engineering, Chang Gung University
259 Wen-Hwa 1st Road, Kwei-Shan, Taoyuan 333, Taiwan

Abstract—This study presents a dual-band rat-race coupler featuring two controllable center frequencies with arbitrary power-split ratios. Novel dual-band stub-loaded 90° lines were used to replace the conventional single-band 90° lines according to the required branch impedances for specific power-split ratios. The proposed structure has two degrees of freedom, providing more design flexibility and obtaining short lengths and realizable impedances. This research successfully synthesized a 2.4/5.2 GHz rat-race coupler with a 1:2 power-split ratio at dual bands. The measured results were well-correlated with simulation results.

I. INTRODUCTION

Nonuniformly excited antenna arrays have shaped the beam and controllable side lobes that are in high demand in modern communication systems. Couplers are often required in the design of feeding networks for providing adequate power distribution to array elements. Rat-race couplers are one of the more attractive candidates. Conventional rat-race couplers require structures with a uniform impedance and specific wavelength; thus, they can be used for only single-band applications. Recently, dual-band antennas have become widely applied in Global Positioning System (GPS) devices and other wireless communication systems. Therefore, the design of dual-band rat-race couplers possessing arbitrary power-split ratios still makes their application an ongoing challenge.

Several approaches to the design of dual-band rat-race couplers have been proposed [1-7]. The λ/4 sections of a conventional rat-race were replaced with composite right/left-handed transmission lines, allowing dual-band operation [1], [2]. These circuits suffered from complex layouts with lumped elements and via holes. In [3], a rat-race coupler operating at two frequencies was devised using tri-section branches and shunt stubs, evaluated according to the average value of lower and upper bands. Another structure using two shunt stubs attached to the ends of a stepped-impedance line was presented in [4]. T-shaped lines were introduced to the branches of a conventional rat-race coupler for dual band operation with a wide frequency ratio [5]. Aflaki et al. [6] designed a rat-race for higher frequency operation using distributed lines, with LC lumped-type resonators connected to open stubs added to the initial design, to enable lower band operation. In [7], dual-band stub-loaded microstrips were utilized to improve coupler performance for equal division of power. The studies of [8] and [9] showed that rat-race couplers can provide adjustable output power divisions by controlling the characteristic impedance values of the λ/4 and 3λ/4 sections, through only for a single band. This study proposes a novel dual-band 90° line to develop a rat-race with controllable dual center frequencies and arbitrary power-split ratios.

II. DESIGN OF DUAL-BAND RAT-RACE COUPLER WITH ARBITRARY POWER-SPLIT RATIOS

Designers typically construct 3 dB rat-race couplers using λ/4 and 3λ/4 lines with an impedance of $\sqrt{2} Z_0$ (or 70.7 $\Omega$). When an arbitrary power division is desired, the uniform impedance must be changed to stepped-impedance values, $Z_a$ and $Z_b$, as shown in Fig. 1(a). One may choose an impedance pair of $Z_a$ and $Z_b$ to achieve various power-split ratios. However, these lines have an electrical length of 90° or 270° at only a single frequency, permitting only single-band performance. Figure 1(b) presents a schematic of the proposed dual-band rat-race coupler with arbitrary power-split ratios, implemented using dual-band 90° lines (marked as section-α and section-β), which is loaded with open-ended stepped-impedance stubs at the middle of the signal paths. The proposed dual-band line is capable of simultaneously providing an equivalent electrical length of 90° and desired impedances at two center frequencies because of its attached shunt susceptance. Three dual-band 90° lines (section-β) are used to replace the 3λ/4 $Z_0$ line in Fig. 1(b).

When a power-split ratio is specified, (1a) and (1b) can be used simultaneously to determine the required coupling coefficients for output ports 2 and 3. Table I lists the coupling coefficients of $|S_{21}|$ and $|S_{31}|$ in dB for power-split ratio from 1:1 to 1:8 when $P_{21} > P_{31}$. The coupling coefficient $|S_{31}|$ can be used to determine the branch impedance of $Z_a$ and $Z_b$ for the desired power-split ratio $P_{31}/P_{21}$ from (1c) and (1d), respectively. Table I also lists the values of $Z_a$ and $Z_b$ for the power-split ratio 1:1-1:8, respectively.
where

\[ \frac{Z_1}{Z_2} = \frac{S_{11}^2}{S_{21}^2} \]  

(1a)

\[ |S_{11}|^2 + |S_{21}|^2 = 1 \]  

(1b)

\[ Z_a = Z_b/\sqrt{1-|S_{11}|^2} \]  

(1c)

\[ Z_b = Z_a |S_{11}| \]  

(1d)

Table I provides data only for single-band rat-race couplers. When a dual-band coupler is designed, the impedances and lengths at dual center frequencies are added to the conventional dual-band line with characteristic impedance values. Generally, smaller power-split ratios and design freedoms are needed for dual-band lines.

Table 

| \( P_{11}/P_{21} \) | \( 20 \log (|S_{11}|) \) (dB) | \( 20 \log (|S_{21}|) \) (dB) | \( Z_1 \) (Ω) | \( Z_2 \) (Ω) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1:1             | -3              | -3              | 70.7            | 70.7            |
| 1:2             | -8.47           | -1.76           | 3.01            | 61.24           |
| 1:3             | -6.02           | -1.25           | 4.77            | 57.74           |
| 1:4             | -6.99           | -0.97           | 6.02            | 55.93           |
| 1:5             | -7.78           | -0.79           | 6.99            | 54.77           |
| 1:6             | -8.45           | -0.67           | 7.78            | 54.01           |
| 1:7             | -9.03           | -0.58           | 8.43            | 53.45           |
| 1:8             | -9.54           | -0.51           | 9.03            | 53.03           |

Rat-race specifications \( \theta_r \) and power-split ratio \( r \) are determined. Calculate \( Z_a \) and \( Z_b \) from (1a)-(1d) and select \( \theta_r = 0.8 \) for realizability and shorter lengths. Solve \( \theta_1, \theta_2, \theta_b, \) and \( \theta_c \) for realizability and section-\( a \) and section-\( b \). As shown in Figs. 3(a) and 3(b), \( Z_1 \) and \( Z_2 \) increase in conjunction with \( r \) for section-\( a \) and section-\( b \). Selection of \( Z_{ab} \) and \( Z_{bc} \) depends on \( Z_1 \) and \( Z_2 \), which must be realizable using the general microstrip fabrication process, typically 20-120 Ω. The range of \( r \) is limited to 0.4 \( \leq r \leq 1.05 \) for section-\( a \), and 0.3 \( \leq r_{ab} \leq 0.87 \) for section-\( b \). Figure 5 shows the applicable range of power-split ratio.

When a dual-band coupler is designed, the impedances and lengths at dual center frequencies are added to the conventional dual-band line with characteristic impedance values. Generally, smaller power-split ratios and design freedoms are needed for dual-band lines. As shown in Figs. 3(a) and 3(b), \( Z_1 \) and \( Z_2 \) increase in conjunction with \( r \) for section-\( a \) and section-\( b \). Selection of \( Z_{ab} \) and \( Z_{bc} \) depends on \( Z_1 \) and \( Z_2 \), which must be realizable using the general microstrip fabrication process, typically 20-120 Ω. The range of \( r \) is limited to 0.4 \( \leq r \leq 1.05 \) for section-\( a \), and 0.3 \( \leq r_{ab} \leq 0.87 \) for section-\( b \).

Table II provides data only for single-band rat-race couplers. When a dual-band coupler is designed, the impedances and lengths at dual center frequencies are added to the conventional dual-band line with characteristic impedance values. Generally, smaller power-split ratios and design freedoms are needed for dual-band lines.

Figure 3(a) shows the design curves of electrical lengths for section-\( a \) and section-\( b \), respectively. As shown in Figs. 3(a) and 3(b), \( Z_1 \) and \( Z_2 \) increase in conjunction with \( r \) for section-\( a \) and section-\( b \). Selection of \( Z_{ab} \) and \( Z_{bc} \) depends on \( Z_1 \) and \( Z_2 \), which must be realizable using the general microstrip fabrication process, typically 20-120 Ω. The range of \( r \) is limited to 0.4 \( \leq r \leq 1.05 \) for section-\( a \), and 0.3 \( \leq r_{ab} \leq 0.87 \) for section-\( b \). Figure 5 shows the applicable range of power-split ratio.

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III. ILLUSTRATIVE EXAMPLE

This study designed an experimental dual-band rat-race coupler to operate at 2.4/5.2 GHz with a power-split ratio of 1:2. By choosing \( r_Z = 0.6 \) and \( r_L = 0.8 \), the coupler was shown to be able to possess adequate impedance values and a short stub length. The circuit dimensions are \( Z_1 = 39 \, \Omega \), \( \theta_1 = 49.05^\circ \), \( Z_2 = 65 \, \Omega \), \( \theta_2 = 61.31^\circ \), \( Z_3 = 40 \, \Omega \), and \( \theta_3 = 113.67^\circ \) for section-\( \alpha \), and \( Z_1 = 55.8 \, \Omega \), \( \theta_1 = 49.05^\circ \), \( Z_2 = 93 \, \Omega \), \( \theta_2 = 61.31^\circ \), \( Z_3 = 56.6 \, \Omega \), and \( \theta_3 = 113.67^\circ \) for section-\( \beta \). Table II lists the design values of dual-band couplers for power-split ratios from 1:1 to 1:8.

Figures 6(a) and 6(b) plot the simulated and measured responses of \(|S_{ij}|\), respectively. The simulated insertion losses are \(|S_{11}| = -1.91 \, \text{dB} \) and \(|S_{31}| = -5.32 \, \text{dB} \) (\( \Delta = 3.41 \, \text{dB} \)) at 2.4 GHz, and \(|S_{21}| = -2.37 \, \text{dB} \) and \(|S_{31}| = -5.12 \, \text{dB} \) (\( \Delta = 2.75 \, \text{dB} \)) at 5.2 GHz, where \( \Delta = |S_{21}|-|S_{31}| \) denotes the power ratio (ideally 3 dB).

The measured results at 2.38 GHz are \(|S_{11}| = -15.1 \, \text{dB} \), \(|S_{21}| = -2.11 \, \text{dB} \), \(|S_{31}| = -5.6 \, \text{dB} \), \(|S_{41}| = -31.3 \, \text{dB} \), and \( \Delta = 3.49 \, \text{dB} \), whereas at 5 GHz \(|S_{11}| = -30.9 \, \text{dB} \), \(|S_{21}| = -2.67 \, \text{dB} \), \(|S_{31}| = -5.68 \, \text{dB} \), \(|S_{41}| = -25.63 \, \text{dB} \), and \( \Delta = 3.01 \, \text{dB} \). Measurements correlated well with the simulation results, except for a slight shift in frequency. The measured bandwidths of the first and second bands were 15\% and 5\%, respectively, defined with \( \Delta \pm 0.5 \, \text{dB} \). Figures 6(c) and 6(d) show the phase differences \( \angle S_{31} - \angle S_{12} \) and \( \angle S_{34} - \angle S_{12} \), respectively. The measured data show that \( \angle S_{31} - \angle S_{12} \) was 1.25\° and 1.54\° and \( \angle S_{34} - \angle S_{12} \) was 180.64\° and -182.31\° at 2.38 GHz and 5 GHz, respectively. The bandwidths of phase difference for \( \angle S_{31} - \angle S_{12} \) were 16\% and 5\%, and the bandwidths of \( \angle S_{34} - \angle S_{12} \) were 180\°± 5\° were 7\% and 5\%, respectively. Figure 6(d) also shows a photograph of the fabricated coupler.
This study developed a dual-band rat-race coupler with arbitrary power division ratios. The impedance pair of $Z_a$ and $Z_b$ was calculated to achieve the desired power-split ratio. Using the proposed dual-band 90° lines, section-$\alpha$ and section-$\beta$, instead of the conventional single-band 90° lines enabled dual-band rat-race performance. The proposed circuit structure provides more design flexibility, yielding short lengths and realizable impedances. An experimental 2.4/5.2 GHz rat-race coupler with a 1:2 power-split ratio was fabricated for demonstration purposes. Measurements showed excellent performance. Although, only one coupler with a power split of 1:2 was presented, the structure of the proposed circuit could be applied to for a variety of power divisions and center frequency ratios.

### Table II

**DESIGN VALUES OF DUAL-BAND RAT-RACE COUPLERS FOR VARIOUS TYPICAL POWER-SPLIT RATIOS (WITH $\beta_1 = 2.167$, $\beta_2 = 0.6$, and $\beta_3 = 0.8$)**

<table>
<thead>
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<th>$\beta_2$</th>
<th>1:2</th>
<th>1:3</th>
<th>1:4</th>
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<tr>
<td>$Z_a(\Omega)$</td>
<td>35.79</td>
<td>80.03</td>
<td>35.29</td>
</tr>
<tr>
<td>$Z_b(\Omega)$</td>
<td>49.82</td>
<td>111.8</td>
<td>46.77</td>
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<tr>
<td>$\theta_1^\circ$</td>
<td>51.7</td>
<td>51.7</td>
<td>51.7</td>
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<tr>
<td>$\theta_2^\circ$</td>
<td>62.82</td>
<td>61.31</td>
<td>61.31</td>
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<tr>
<td>$Z_0(\Omega)$</td>
<td>29.89</td>
<td>67.08</td>
<td>28.06</td>
</tr>
</tbody>
</table>

IV. CONCLUSIONS

Fig. 6. Experimental dual-band rat-race coupler with 1:2 power-split ratio: (a) Simulated $|S_{ij}|$, (b) Measured $|S_{ij}|$, (c) $\angle S_{11}$, $\angle S_{12}$, (d) $\angle S_{21}$, $\angle S_{34}$.

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REFERENCES